## MECHANICS.

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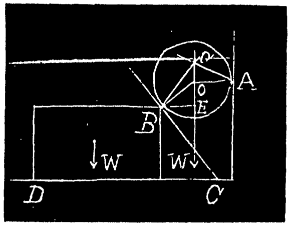
**82. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.**

A sphere, diameter $2a$. rests in limiting equilibrium upon the edge of a box and against a vertical wall. If the box be of such dimensions that it will not tip, find the distance of the box from the wall, having given the coefficient of friction between the sphere and wall $\frac{1}{2}$, between the sphere and box $\frac{1}{3}$, and between the box and floor $\frac{2}{3}$. [From Problems in Mechanics proposed to class in Harvard University.]

**Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Let $W$=weight of sphere, $W'$=weight of box, $\mu=\frac{1}{2}$, $\mu'=\frac{1}{3}$, $\mu''=\frac{2}{3}$, $\theta=\angle BCD$, $S$=normal reaction of wall, $R$=normal reaction of box, $d$=distance of box from wall.

$\therefore\ d=AO+BE=a(1+\sin\theta)$, since $BO$ is perpendicular to $BC$.




Also $S=\mu' R\cos\theta+R\sin\theta=\mu'' W'$ (resolving horizontally).

$$\therefore\ S=\tfrac{2}{3}W',\ R=\frac{\mu'' W'}{\mu'\cos\theta+\sin\theta}$$

$$=\frac{2W'}{\cos\theta+3\sin\theta}.$$

Also $\mu S+\mu' R\sin\theta+R\cos\theta=W$ (resolving vertically), or $\frac{1}{2}S+\frac{1}{3}R\sin\theta+R\cos\theta=W$.

The values of $S$ and $R$ in the last equation give

$$\tfrac{1}{3}W'+\frac{2W'\sin\theta}{3\cos\theta+9\sin\theta}+\frac{2W'\cos\theta}{\cos\theta+3\sin\theta}=W.$$

$$\therefore\ \tan\theta=\frac{7W'-3W}{9W-5W'},\ \sin\theta=\frac{7W'-3W}{\sqrt{90W^2-132WW'+74W'^2}}.$$

$$\therefore\ d=a\left[\frac{\sqrt{90W^2-132WW'+74W'^2}+7W'-3W}{\sqrt{90W^2-132WW'+74W'^2}}\right].$$

If $W=W'$, $d=\frac{1}{2}a(2+\sqrt{2})$.

**83. Proposed by MARY M. BLAINE, B. Sc., Graduate Student, Drury College, Springfield, Mo.**

A particle is projected upwards in vacuo with a velocity $v$. Show that on reaching the ground again there is no deviation to the south, but the deviation to the west is $4\omega\cos\lambda(v^3/3g^2)$. [*Laplace*, iv, page 341.]

**Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

The equations of motion are

$$\frac{d^2x}{dt^2}+2\omega\sin\lambda\frac{dy}{dt}=0,\ \frac{d^2y}{dt^2}-2\omega\cos\lambda\frac{dz}{dt}-2\omega\sin\lambda\frac{dx}{dt}=0,\ \frac{d^2z}{dt^2}+2\omega\cos\lambda\frac{dy}{dt}=-g.$$

(Routh's *Advanced Rigid Dynamics*, fourth edition, page 20.)

As a first approximation, we can neglect the motion of the earth. Then, from mechanics, $x=0$, $y=0$, $z=vt-\frac{1}{2}gt^2$.

$$\therefore \frac{dx}{dt}=0,\ \frac{dy}{dt}=0,\ \frac{dz}{dt}=v-gt.$$

$$\therefore \frac{d^2x}{dt^2}=0,\ \frac{d^2y}{dt^2}=2\omega\cos\lambda(v-gt).$$

$$\therefore x=0,\ y=\omega\cos\lambda(vt^2-\tfrac{1}{3}gt^3).\quad \text{But } t=2v/g.$$

$$\therefore x=0,\ y=\omega\cos\lambda(4v^3/g^2-8v^3/3g^2)=4\omega\cos\lambda(v^3/3g^2).$$

$\therefore$ deviation south$=0$, west$=4\omega\cos\lambda(v^3/3g^2)$.

## AVERAGE AND PROBABILITY.

67. **Proposed by HENRY HEATON, M. Sc., Atlantic, Ia.**

A witness in court who undertook to recognize the signature of an individual failed four times in succession. What is the probability that he was correct the fifth time? An actual occurrence.

**Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chaster High School, Chester, Pa., and CHARLES CARROLL CROSS, Libertytown, Md.**

Let $p$=chance, $p_1$=chance of failure.

$$\therefore p_1=\frac{\int_0^1 x^5dx}{\int_0^1 x_4dx}=\tfrac{5}{6}.\quad \therefore p=1-\tfrac{5}{6}=\tfrac{1}{6}.$$

68. **Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.**

What are the odds against throwing 7 or 11 at one throw with two dice?

I. **Solution by CHARLES CARROLL CROSS, Libertytown, Md.**

Each dice may fall in any one of 6 ways, therefore, both dice in $6\times6=36$ ways.

$11=6+5$ or $5+6$; hence the chance against throwing 11 at one throw is $1-\frac{2}{36}=\frac{17}{18}$.